

Upper limits on electric dipole moments of τ -lepton, heavy quarks, and W -boson

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Abstract

We discuss upper limits on the electric dipole moments (EDM) of the τ -lepton, heavy quarks, and W -boson, which follow from the precision measurements of the electron and neutron EDM.

1 Introduction

Strict upper limits on the electric dipole moments of common elementary particles, electron and proton, were derived from spectroscopic, almost table-top experiments [1, 2, 3]. As to the neutron EDM, the best upper limit on it was obtained as a result of reactor experiments lasting many years [4] (they say that the searches for the neutron EDM killed more theories than any other experiment in the history of physics). And at last, the result for the muon EDM follows from the measurements at the dedicated muon storage ring [5]. These results are summarized in Table 1.

	e	p	n	μ
d/e , cm	$(0.7 \pm 0.7) \times 10^{-27}$ [1]	$< 0.8 \times 10^{-24}$ [2, 3]	$< 0.29 \times 10^{-25}$ [4]	$(0.37 \pm 0.34) \times 10^{-18}$ [5]

Table 1

As to the dipole moments of the τ -lepton and heavy quarks, upper limits on them have been obtained up to now from the analysis of high-energy experiments.

The approach pursued here is based on the precision results [1, 4]. We establish upper limits on the dipole moments of the τ -lepton and heavy quarks through the analysis of their possible contributions to the electron EDM. This is a clean theoretical problem for the τ -lepton. For heavy quarks and W -boson these our results are of rather qualitative nature. Additional upper limits on the dipole moments of heavy quarks, and W -boson, also qualitative ones, are derived by the analysis of their possible contributions to the neutron EDM.

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2 Dipole moment of τ -lepton and electron EDM

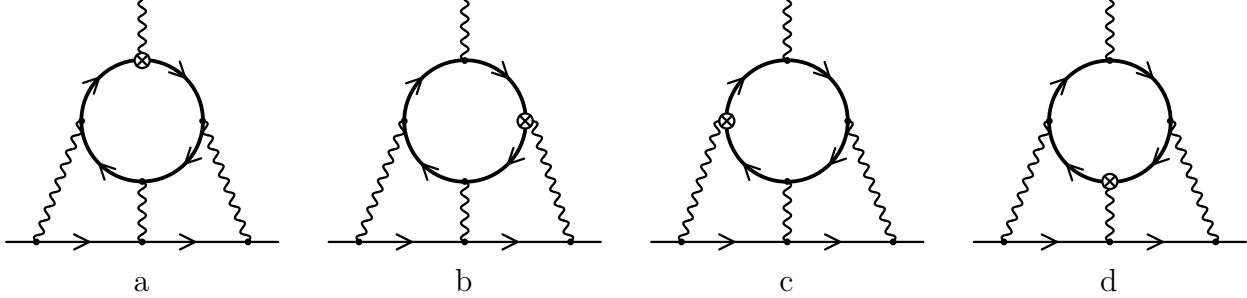


Figure 1

We start with the analysis of the contribution of the τ EDM d_τ to the electron dipole moment d_e . This contribution is described by the diagrams of the type presented in Figs. 1a,b,c,d. Here the loop is formed by the τ line, and the lower solid line is the electron one. The upper wavy line corresponds to the external electric field. The crossed vertices refer to the electromagnetic interaction of the τ EDM

$$L_\tau^{edm} = -\frac{1}{2} d_\tau \bar{\tau} \gamma_5 \sigma_{\mu\nu} \tau F_{\mu\nu} = i \frac{1}{2} d_\tau \bar{\tau} \sigma_{\mu\nu} \tau \tilde{F}_{\mu\nu}; \quad \tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}. \quad (1)$$

Of course, all six permutations of the electromagnetic vertices on the electron line should be considered. The contributions of diagrams 1b and 1c are equal.

This problem is similar to that of the contribution by the light-by-light scattering via muon loop to the electron magnetic moment [6]. The general structure of the resulting contribution to the electron EDM is rather obvious (to the leading order in m_e/m_τ):

$$\Delta d_e = a \frac{m_e}{m_\tau} \left(\frac{\alpha}{\pi} \right)^3 d_\tau, \quad (2)$$

where a is some numerical factor (hopefully, on the order of unity). The factor m_e originates from the necessary helicity-flip on the electron line; then $1/m_\tau$ is dictated by dimensional arguments.

Diagram 1a corresponds to the matrix element $\langle e | \bar{\tau} \sigma^{\mu\nu} \tau | e \rangle = C \bar{u} \sigma^{\mu\nu} u$. We use dimensional regularization with $d = 4 - 2\varepsilon$ dimensions and the method of regions (see the textbook [7]). Only the region where all three loops are hard (loop momenta $\sim m_\tau$) contributes to the leading power term (2); therefore, there are no logarithms $\ln(m_\tau/m_e)$ (contributions of regions with 1 or 2 hard loops are suppressed by an extra factor $(m_e/m_\tau)^2$). In this hard region, the problem reduces to 3-loop vacuum integrals with a single mass m_τ belonging to the simpler topology B_M [8]. We perform the calculation in arbitrary covariant gauge, and use the REDUCE package RECURSOR [8] to reduce scalar integrals to two master integrals. Gauge-dependent terms cancel, and we get

$$C = \frac{m_e}{m_\tau} \frac{e^6 m_\tau^{-6\varepsilon}}{(4\pi)^{3d/2}} \Gamma^3(\varepsilon) \frac{8}{d(d-1)(d-5)} \left[-2 \frac{2d^2 - 21d + 61}{d-5} + \frac{d^4 - 9d^3 + 8d^2 + 84d - 126}{2d-9} R \right], \quad (3)$$

where

$$R = \frac{\Gamma(1-\varepsilon)\Gamma^2(1+2\varepsilon)\Gamma(1+3\varepsilon)}{\Gamma^2(1+\varepsilon)\Gamma(1+4\varepsilon)} = 1 + 8\zeta(3)\varepsilon^3 + \dots, \quad (4)$$

and ζ is the Riemann ζ -function. All divergences cancel, and we arrive at the finite contribution to a :

$$a_1 = \frac{3}{2} \zeta(3) - \frac{19}{12}. \quad (5)$$

In order to calculate the contribution of Fig. 1b,c,d, we expand the corresponding initial expressions in the external photon momentum q up to the linear term. The EDM vertex contains $\varepsilon^{\mu\nu\alpha\beta}$; we put this factor aside, and calculate tensor diagrams with four indices. After summing all diagrams, the result is finite; now we can set $\varepsilon \rightarrow 0$, and multiply by $\varepsilon^{\mu\nu\alpha\beta}$ (cf. [9]). The result has the structure of a tree diagram with the electron EDM vertex $\varepsilon^{\mu\nu\alpha\beta} q_\nu \sigma_{\alpha\beta}$. The gauge-dependent terms in it cancel (exactly in d), as well as the divergences. This contribution to a is

$$a_2 = \frac{9}{4} \zeta(3) - 1. \quad (6)$$

As an additional check of our programs, we have reproduced the leading power term in the contribution to the electron magnetic moment originating from the light-by-light scattering via the muon loop (formula (4) in [6]).

The final result for the numerical coefficient is

$$a = a_1 + a_2 = \frac{15}{4} \zeta(3) - \frac{31}{12} = 1.924. \quad (7)$$

With this value of a , the discussed contribution to the electron EDM is

$$\Delta d_e = 6.9 \times 10^{-12} d_\tau. \quad (8)$$

Combining this result with the experimental one [1] (see Table 1) for the electron EDM, we arrive at

$$d_\tau/e = (1 \pm 1) \times 10^{-16} \text{ cm}. \quad (9)$$

In fact, the results (5) and (6) refer to somewhat different regions of incoming momenta. For (6) all the three momenta are hard, on the order of magnitude about m_τ , but for (5) only two of them belong to this region, and the third one, that of the outer photon, is soft, of vanishing momentum. Still, one may expect that the effective EDM interaction is formed at momenta much higher than m_τ , so that this difference is not of much importance. Besides, the contribution of diagram 1a is anyway numerically small. Thus, result (5) is valid at least for all momenta about $m_\tau \sim 1 - 2 \text{ GeV}$.

The upper limits on the τ EDM derived from the accelerator experiments [10, 11, 12, 13] belong to the interval of $10^{-16} - 10^{-17} e\cdot\text{cm}$, so that our result (9) formally does not improve them. However, all those accelerator data refer to much larger typical momenta of the photon, from 10 to 200 GeV.

3 Dipole moments of heavy quarks and electron EDM

The information on the dipole moments d_q of heavy quarks can be obtained from the diagrams analogous to those presented in Figures 1, but with quarks in the fermion loops, instead of the τ -lepton. We do not consider the gluon corrections to the quark loops, and confine to the simple estimate for them, following from the analogy between the two problems (see (2)):

$$\Delta d_e = a_q 3 Q^3 \frac{m_e}{m_q} \left(\frac{\alpha}{\pi} \right)^3 d_q; \quad (10)$$

here m_q is the quark mass, Q is the quark charge in the units of e ($Q_b = -1/3$, $Q_{c,t} = 2/3$). As to the overall numerical factor here, we put $a_q \simeq 1$. The corresponding estimates are straightforward. Here and below we assume the following values for the quark masses:

$$m_c = 1.25 \text{ GeV}; \quad m_b = 4.5 \text{ GeV}; \quad m_t = 175 \text{ GeV}.$$

Thus obtained upper limits are:

$$d_c/e \lesssim 3 \times 10^{-16} \text{ cm}; \quad d_b/e \lesssim 7 \times 10^{-15} \text{ cm}; \quad d_t/e \lesssim 4 \times 10^{-14} \text{ cm}. \quad (11)$$

4 Dipole moments of heavy quarks and neutron EDM

The dipole moment of a heavy quark Q generates the EDM of a light quark q via the following diagram [14]:

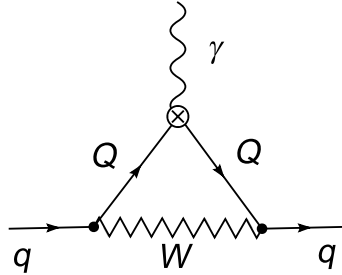


Figure 2

The scattering amplitudes and corresponding contributions to the dipole moments of light quarks are:

$$M_1 = \frac{\alpha}{16\pi \sin^2 \theta_w} |V_{Qq}|^2 \frac{m_q m_Q}{m_W^2} \left(\ln \frac{\Lambda^2}{m_W^2} - 2 \right) \frac{1}{2} d_Q \bar{u} \gamma_5 \sigma_{\mu\nu} u F_{\mu\nu}, \quad (12)$$

$$\Delta d_q = \frac{\alpha}{16\pi \sin^2 \theta_w} |V_{Qq}|^2 \frac{m_q m_Q}{m_W^2} \left(\ln \frac{\Lambda^2}{m_W^2} - 2 \right) d_Q, \quad (13)$$

for

$$Q = c, \quad q = d \quad \text{or} \quad Q = b, \quad q = u;$$

and

$$M_2 = \frac{\alpha}{16\pi \sin^2 \theta_w} |V_{td}|^2 \frac{m_t m_d}{m_W^2} \left(\ln \frac{\Lambda^2}{m_t^2} - 2 \right) \frac{1}{2} d_t \bar{u} \gamma_5 \sigma_{\mu\nu} u F_{\mu\nu}, \quad (14)$$

$$\Delta d_d = \frac{\alpha}{16\pi \sin^2 \theta_w} |V_{td}|^2 \frac{m_t m_d}{m_W^2} \left(\ln \frac{\Lambda^2}{m_t^2} - 2 \right) d_t, \quad (15)$$

for

$$Q = t, \quad q = d.$$

Here V_{Qq} are the corresponding coefficients of the Kobayashi-Maskawa matrix; θ_w is the Weinberg angle, $\sin^2 \theta_w = 0.23$. The structures of relations (13) and (15) are different since $m_{c,b} \ll m_W$, while $m_t > m_W$. In fact, to simplify final expressions, we confine to the leading order in $1/m_W^2$ in (12), (13), and even assume that $m_t \gg m_W$ in (14), (15). In both cases

the diagrams are logarithmically divergent, so that we have to introduce a cut-off Λ at high momenta.

This divergence is caused by the term $k_\mu k_\nu / m_W^2$ in the numerator of the W Green function

$$\frac{-\delta_{\mu\nu} + k_\mu k_\nu / m_W^2}{k^2 - m_W^2}.$$

The mentioned term $k_\mu k_\nu / m_W^2$ was omitted in [14]. Therefore, relations (13) and (15) differ essentially from the corresponding results of [14].

In our estimates here, we assume that both $\ln(\Lambda^2/m_W^2) - 2$ and $\ln(\Lambda^2/m_t^2) - 2$ are on the order of unity. As to the light quark dipole moments, we assume (in the spirit of the constituent quark model) that they are on the same order of magnitude as the neutron EDM:

$$d_{u,d} \sim d_n.$$

Of course, both these assumptions make the corresponding estimates less definite than those based on electron EDM. The results of our estimates are presented in the last line of Table 2. In its previous line we repeat for comparison the data already given in (11). In both sets of estimates the strong interaction of quarks is neglected.

	c	b	t
m_Q , GeV	1.25	4.5	175
d_Q/e , cm derived with d_e	$\lesssim 3 \times 10^{-16}$	$\lesssim 7 \times 10^{-15}$	$\lesssim 4 \times 10^{-14}$
d_Q/e , cm derived with d_n	$\lesssim 10^{-15}$	$\lesssim 2 \times 10^{-12}$	$\lesssim 5 \times 10^{-15}$

Table 2. Limits on quark dipole moments from electron and neutron EDMs

It is only natural that for the most heavy t -quark the bound from d_n dominates.

The bounds on the dipole moments of c - and b -quarks derived from the high-energy experiments [13, 15] are on the order of 10^{-17} e·cm, so that they are more strict than the limits given in Table 2. However, as noted above already, the high-energy data refer to different, much larger typical momenta of the photon.

In conclusion of this section, we mention the investigation of the radiative decay $b \rightarrow s\gamma$ as a probe of the t -quark EDM [16]. The discussed contribution to the decay amplitude is described by the diagram in Fig. 3. This diagram is also logarithmically divergent. Its contribution can be easily obtained as follows. In the limit $m_b \rightarrow m_s$, the structure of discussed matrix element should coincide with that of (14) (up to the obvious changes $m_d \rightarrow m_{b(s)}$ and $|V_{td}|^2 \rightarrow V_{ts}^* V_{tb}$). On the other hand, in the limit $m_s \rightarrow 0$, the outgoing s -quark should be left-handed. Therefore, the matrix element of the $b \rightarrow s\gamma$ decay (with the internal t -quark) should look as follows:

$$M(b \rightarrow s\gamma) = \frac{\alpha}{64\pi \sin^2 \theta_w} V_{ts}^* V_{tb} \frac{m_t}{m_W^2} \left(\ln \frac{\Lambda^2}{m_t^2} - 2 \right) d_t \bar{u}_s \left[-m_b(1 - \gamma_5) + m_s(1 + \gamma_5) \right] \sigma_{\mu\nu} u_b F_{\mu\nu}$$

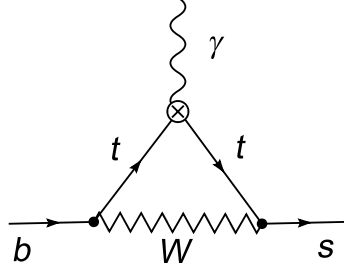


Figure 3

$$\simeq -\frac{\alpha}{64\pi \sin^2 \theta_w} V_{ts}^* V_{tb} \frac{m_b m_t}{m_W^2} \left(\ln \frac{\Lambda^2}{m_t^2} - 2 \right) d_t \bar{u}_s (1 - \gamma_5) \sigma_{\mu\nu} u_b F_{\mu\nu}. \quad (16)$$

The result presented in [16] is

$$d_t/e \lesssim 10^{-16} \text{ cm}. \quad (17)$$

5 Electric dipole moment of W -boson

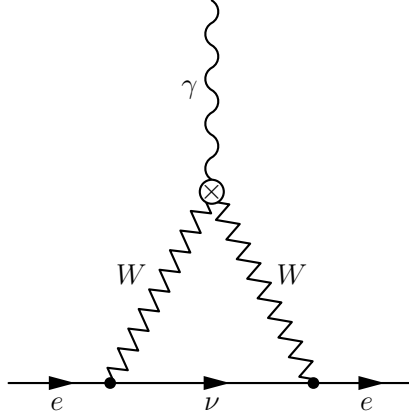


Figure 4

One more contribution to the electron and neutron dipole moments can be given by the EDM d_W of W -boson. This effect was pointed out and investigated long ago [17, 18].

It is convenient to start the discussion with the electron EDM. The effect is described by diagram presented in Fig. 4. Here as well the crossed vertex refers to the electromagnetic interaction of the W -boson EDM (it is obvious from the diagram that here it is the dipole moment of W^-). In this case the EDM interaction is described by the Lagrangian

$$L_W^{edm} = 2m_W i d_W \tilde{F}_{\alpha\beta} W_\alpha^\dagger W_\beta. \quad (18)$$

The corresponding matrix element is

$$M = \frac{\pi\alpha}{\sin^2 \theta_w} d_W m_W \int \frac{d^4 k}{(2\pi)^4} \bar{u}_e(p) \gamma_\mu (1 + \gamma_5) \frac{\hat{k}}{k^2} \gamma_\nu (1 + \gamma_5) u_e(p) \times \\ \times \frac{1}{[(k-p)^2 - m_W^2]^2} \left\{ \tilde{F}_{\mu\nu} - \frac{1}{m_W^2} \left[(k-p)_\mu (k-p)_\alpha \tilde{F}_{\alpha\nu} + \tilde{F}_{\mu\alpha} (k-p)_\alpha (k-p)_\nu \right] \right\}. \quad (19)$$

With straightforward, though rather tedious calculations (somewhat simplified by employing the density matrix of polarized fermion), one arrives at the following result for the contribution of W -boson EDM to electron dipole moment:

$$\Delta d_e = \frac{\alpha}{8\pi \sin^2 \theta_w} \frac{m_e}{m_W} \ln \frac{\Lambda^2}{m_W^2} d_W. \quad (20)$$

Here Λ is the cut-off parameter for the logarithmically divergent integral over virtual momenta in the loop. Putting (perhaps, quite conservatively) $\ln(\Lambda^2/m_W^2) \simeq 1$, one obtains with the experimental upper limit on the electron EDM [1] (see Table 1), the following bound on the dipole moment of W -boson:

$$d_W/e \lesssim 2 \times 10^{-19} \text{ cm}. \quad (21)$$

In the case of the W -boson contribution to the neutron EDM, our line of reasoning somewhat differs from that of [17, 18]. We note first of all that the electron mass m_e does not enter explicitly matrix element (19). It arises in the result (20) only as the mass of an external fermion, via the Dirac equation $\hat{p}u = m_e u$. Therefore, there are all the reasons to expect that the contribution of d_W to the neutron EDM will be proportional to the neutron mass m_n , i.e. enhanced as compared to (20) by three orders of magnitude. In this case, the forward scattering amplitude of the virtual W -boson can be written in a general form as follows⁴:

$$\bar{u}_n(p) \gamma_\mu (1 + \gamma_5) \left[\hat{k} g(k^2) + \hat{p} h(k^2) \right] \gamma_\nu (1 + \gamma_5) u_n(p); \quad (22)$$

here k is the total momentum of intermediate hadronic states. Of course, the invariant functions g and h depend in fact not only on k^2 , but on (kp) as well. However, in our case $k^2 \sim m_W^2 \gg (kp) \sim m_n m_W$, so that the dependence on (kp) can be safely ignored. By the analogous reason, in the usual case of the deep inelastic neutrino scattering, the structure with \hat{p} in the corresponding amplitude is also omitted. In the present case, however, we should keep in amplitude (22) \hat{p} , in addition to the common \hat{k} , since after integrating over d^4k both structures give comparable contributions to the result.

At last, the usual dimensional and scaling arguments dictate that asymptotically, for $k^2 \sim m_W^2$, both functions g and h behave as follows:

$$g(k^2) = \frac{g_0}{k^2}, \quad h(k^2) = \frac{h_0}{k^2}.$$

In particular, one can neglect the gluon corrections in these functions. Without any additional parameters, it is natural to assume that $g_0, h_0 \sim 1$.

Now, the same calculations as those in the case of electron EDM, result in the following expression for the discussed contribution to the neutron dipole moment:

$$\Delta d_n = \frac{\alpha}{8\pi \sin^2 \theta_w} \frac{m_n}{m_W} \left[g_0 \ln \frac{\Lambda^2}{m_W^2} + h_0 \left(\ln \frac{\Lambda^2}{m_W^2} + 1 \right) \right] d_W. \quad (23)$$

For numerical estimate we put

$$g_0 \ln \frac{\Lambda^2}{m_W^2} + h_0 \left(\ln \frac{\Lambda^2}{m_W^2} + 1 \right) \sim 1,$$

so that

$$\Delta d_n \sim \frac{\alpha}{8\pi \sin^2 \theta_w} \frac{m_n}{m_W} d_W \approx \frac{\alpha}{2\pi} \frac{m_n}{m_W} d_W.$$

⁴Compare with the corresponding structure $\bar{u}_e(p) \gamma_\mu (1 + \gamma_5) (\hat{k}/k^2) \gamma_\nu (1 + \gamma_5) u_e(p)$ in formula (19) for electron.

Then, with the result of [4] for the neutron EDM (see Table 1), we arrive at the following quite strict upper limit on the W -boson dipole moment:

$$d_W/e \lesssim 2 \times 10^{-21} \text{ cm} . \quad (24)$$

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